

MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #11

Problem 1 (DF 14.2.14). Let $K = \mathbb{Q}(\sqrt{2 + \sqrt{2}})$.

- (a) Show that $[K : \mathbb{Q}] = 4$.
- (b) Show that K is Galois and $\text{Gal}(K/\mathbb{Q}) \cong \mathbb{Z}/4\mathbb{Z}$.

Problem 2. Let K/\mathbb{Q} be a finite extension.

- (a) Prove that $\#\text{Hom}(K, \mathbb{C}) = [K : \mathbb{Q}]$. [*Hint: Use induction on $[K : \mathbb{Q}]$, and follow the proof from class.*]
- (b) Let L/K be a finite extension. Prove that every embedding $K \hookrightarrow \mathbb{C}$ extends to $[L : K]$ embeddings of $L \hookrightarrow \mathbb{C}$. [*Hint: Modify the proof as in (a).*]