MATH 351: RIEMANN SURFACES AND DESSINS D'ENFANTS
HOMEWORK #2

Problem 2.1. Let X be a topological space.

(a) Let $Z \subseteq X$. Let $\mathcal{V} = \{U \cap Z : U \text{ open in } X\}$. Show that $\mathcal{V}$ defines a topology on Z, called the *subspace* (or *relative*) *topology* on Z. Show that $\mathcal{V}$ is the smallest topology on Z such that the inclusion map $Z \hookrightarrow X$ is continuous.

(b) Now suppose that $f : X \to Y$ is a homeomorphism. Show that $f|_Z : Z \to f(Z)$ is a homeomorphism, if Z and $f(Z)$ are given the subspace topology.

Problem 2.2. Let $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$ be the cone in $\mathbb{R}^3$ with the subspace topology. Show that X cannot be given the structure of a topological surface.

Date: Wednesday, 16 January 2013.