

**MATH 351: RIEMANN SURFACES AND DESSINS D'ENFANTS  
HOMEWORK #6**

**Problem 6.1.** Let  $X$  be a Riemann surface. Show that meromorphic functions on  $X$  are the same as morphisms  $X \rightarrow \mathbb{P}^1(\mathbb{C})$ , with one exception: prove that

$$\mathbb{C}(X) = \text{Hom}(X, \mathbb{P}^1(\mathbb{C})) \setminus \{\infty\}$$

where

$$\begin{aligned} \infty : X &\rightarrow \mathbb{P}^1(\mathbb{C}) \\ x &\mapsto \infty \end{aligned}$$

is the constant map.

*[Hint: This is Proposition 1.23 in the book. Explain the proof in a level of detail suitable to your understanding (e.g., review removable singularities).]*