MATH 351: RIEMANN SURFACES AND DESSINS D'ENFANTS
HOMEWORK #8

Problem 8.1. Show that

$$g(X_1 \# X_2) = g(X_1) + g(X_2)$$

for compact, orientable (triangulable) surfaces X_1, X_2. Conclude (informally) that "the genus of an compact orientable surface is the number of holes."

Problem 8.2. Let X be a triangulation of a compact surface with v vertices, e edges, and f faces, and let $\chi(X) = v - e + f$. Show that:

(a) $2e = 3f$.
(b) $e = 3(v - \chi)$.
(c) $e \leq v(v-1)/2$, hence

$$v \geq \frac{1}{2}\left(7 + \sqrt{49 - 24\chi}\right).$$

Conclude that for the sphere ($\chi(\mathbb{S}^2) = 2$), the triangulation with the smallest number of faces is the tetrahedron, and for the torus ($\chi(\mathbb{T}^2) = 0$) any triangulation has at least 14 faces.

Problem 8.3. Use the fact that $\chi(\mathbb{S}^2) = 2$ to show that there are only five regular polyhedra. *[Hint: Consider subdivisions of the sphere into n-gons such that exactly m edges meet at each vertex, $m, n \geq 3$. Show that $nf = 2e = mv$, so that*

$$\frac{1}{e} + \frac{1}{2} = \frac{1}{m} + \frac{1}{n}$$

and consider the solutions with $n = 3, 4, 5$ and $n \geq 6$ in turn.]

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