MATH 351: RIEMANN SURFACES AND DESSINS D'ENFANTS
HOMEWORK #11

Problem 11.1. Let $f(z) \in \operatorname{Aut}(\mathbb{C})$ be an automorphism, so that $f(z) = az+b$ with $a, b \in \mathbb{C}$ with $a \neq 0$. Show that f is an isometry of $\mathbb{C}$ under the metric $ds^2 = |dz|^2 = dx^2 + dy^2$ if and only if $|a| = 1$. *[Hint: $|df(z)| = |df/dz||dz|$.]*

Problem 11.2. Let $f(w) \in \operatorname{Aut}(\mathbb{P}^1)$ be an automorphism with $f(\infty) = \infty = [1 : 0]$, so that $f(w) = aw + b$. Show that f is an isometry of $\mathbb{P}^1 = \mathbb{S}^2$ (with the spherical metric $ds^2 = dx^2 + dy^2 + dz^2$) if and only if $|a| = 1$ and $b = 0$, in which case f is a rotation about the z axis. *[Hint: Argue with the metric; or be clever and use the fact that $d(0, \infty) = d(b, \infty)$ to conclude that $b = 0$ and then $d(0, 1) = d(0, a)$ to conclude $|a| = 1$.]*

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