

**MATH 351: RIEMANN SURFACES AND DESSINS D'ENFANTS
HOMEWORK #12**

Problem 12.1. The upper half-plane \mathbb{H} has the metric

$$ds = \frac{|dz|}{\operatorname{Im} z}.$$

Via the map

$$\begin{aligned}\phi : \mathbb{H} &\rightarrow \mathbb{D} \\ z &\mapsto w = \frac{z - i}{z + i}\end{aligned}$$

there is an induced metric on \mathbb{D} . Show that this metric is

$$ds = \frac{2|dw|}{1 - |w|^2}.$$

[Hint: If $\psi = \phi^{-1}$, the induced metric is

$$\frac{|d\psi(w)|}{\operatorname{Im} \psi(w)}.]$$