MATH 351: RIEMANN SURFACES AND DESSINS D'ENFANTS
HOMEWORK #23

Let X be a compact, connected Riemann surface.

Problem 23.1. Let $\omega \in K^1(X)$ be a meromorphic differential on X and $f \in \mathbb{C}(X)$ a meromorphic function. Show that $f\omega$ is a meromorphic differential. Conclude that $K^1(X)$ is a one-dimensional vector space over the field $\mathbb{C}(X)$.

Problem 23.2. Let $f \in \mathbb{C}(X)$ is a meromorphic function and $\omega \in K^1(X)$ a meromorphic differential.

(a) Show that if $m_p(f) = m \neq 0$ then $\mathrm{ord}_p(df) = m - 1$.
(b) Show that if $\mathrm{ord}_p(\omega) = k$ and $m_p(f) = m$ then $\mathrm{ord}_p(f\omega) = m + k$.

Now let $F(x, y, z) = y^2 z - p(x, z)$ where $p(x, z) = x^3 + Axz^2 + Bz^3$ and let $X = Z(F) \subseteq \mathbb{P}^2(\mathbb{C})$ be the projective plane curve associated to F. Show that dx/y is a holomorphic differential at $\infty = [0 : 1 : 0]$ as follows.

(c) Show that x has a double pole at ∞ and y a triple pole at ∞.
(d) Conclude from (a) and (b) that $\mathrm{ord}_\infty(dx/y) = 0$.

Date: Wednesday, 13 March 2013.