MATH 115: ELEMENTARY NUMBER THEORY
HOMEWORKS # 1–2

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Homework #1 (Due June 28):

- §1.1: 4, 5, 8, 9;
  1.1A: A set $S \subset \mathbb{R}$ is *well-ordered* if for every subset $T \subset S$, T has a *least element.*
  (a) Show that the sets
  $$\mathbb{Z}_{<0} = \{-1, -2, -3, \dots\}$$
  and
  $$\{1/n : n \in \mathbb{Z}_{>0}\} = \{1, 1/2, 1/3, 1/4, 1/5, \dots\}$$
  are not well-ordered.
  (b) Show that if $S \subset \mathbb{R}$ is well-ordered, then S cannot contain a decreasing sequence of distinct real numbers.
- §1.4: 6, 7, 8, 17, 18, 36, 46
- §3.1: 6, 7, 10, 11, 16;
  3.1A: Let $p_1 = 2, p_2 = 3, \dots$ be the sequence of increasing primes, so that p_n is the nth prime. Is the number $P_n = p_1 p_2 \cdots p_n + 1$ itself always prime?
  3.1B: For $x \in \mathbb{R}_{>0}$, let $s(x)$ denote the number of positive square integers not exceeding x, namely
  $$s(x) = \#\{n^2 \leq x : n \in \mathbb{Z}_{>0}\}$$
  Show that $s(x) \sim \sqrt{x}$.
- §3.2: 5, 6, 8, 9, 16

Homework #2 (Due July 6, no class Monday, July 5):

- §3.3: 1(a), 1(d), 3(a), 3(d), 9;
  3.3A: Let $f_0 = 1$, $f_1 = 1$, $f_{i+1} = f_i + f_{i-1}$ (for $i > 0$) be the sequence of Fibonacci numbers. Prove that $f_i < f_{i+2}/2$ for all $i > 0$. *[Hint: Use Theorem 3.11 or prove it directly.]*
  3.3B: Assume that the limit
  $$\alpha = \lim_{i \to \infty} \frac{f_{i+1}}{f_i}$$
  exists. Show that $\alpha = (1 + \sqrt{5})/2$. *[This explains where α comes from in Example* 1.24.*]*
- §3.4: 2, 4(d), 7, 10, 19–22, 39, 44, 47
- §4.1: 4, 8, 16, 17, 20, 26
- §3.6: 1(a)–(c);
  3.6A: Find all integers $x, y \in \mathbb{Z}$ for which
  $$x^2 = 4y^2 + 9.$$