

**MATH 115: ELEMENTARY NUMBER THEORY  
HOMEWORK #5 WORKSHEET**

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First, break up into the assigned groups, introduce yourselves, and then please write the names of all of your group members on the board. Then write up a nice solution to your assigned problem on the board. At the end of class, any member of your group may be called upon to explain your solution, so make sure everyone understands! (If your group finishes early, you might think about one of the other problems.)

(1) Show that there is no positive integer  $n$  such that  $\phi(n) = 14$ .

(2) Show that if  $n \in \mathbb{Z}_{>1}$ , then

$$\prod_{p|n} p \geq \frac{n}{\phi(n)}.$$

(3) Let  $k$  be a positive integer. Show that if the equation  $\phi(n) = k$  has exactly one solution  $n$ , then  $36 \mid n$ .

(4) Find all positive integers  $n$  with  $\sigma(n) = 24$ .

(5) Factor  $n = 2^{30} - 1$  completely.