MIDTERM EXAM REVIEW
MATH 115: NUMBER THEORY

Problem 1. Show that the set $S \subset \mathbb{R}$ of positive irrational numbers is not well-ordered.

Problem 2. Find a solution $x \in \mathbb{Z}/65\mathbb{Z}$ to

$$x^2 + 1 \equiv 0 \pmod{65}.$$

How many distinct solutions $x \in \mathbb{Z}/65\mathbb{Z}$ are there to this congruence?

Problem 3. Show that if p, q are distinct primes, then

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}.$$

Problem 4. An integer x is randomly chosen between 100 and 1000. Estimate the probability that x is prime. *[Hint:* $\log(10) \approx 2.5$.*]*

Problem 5. For which integers n is it true that $n - 2$ divides $2n^2 - 1$?

Problem 6. Find a solution $x \in \mathbb{Z}$ to the congruence

$$x^2 + 1 \equiv 0 \pmod{101^2}.$$

[Hint: 101 *is prime.]*