MIDTERM EXAM
MATH 115: NUMBER THEORY

Answer each question completely, and give sufficient justification and proof. Write neatly and in complete sentences!

Name	
Student ID	

Problem 1	/10
Problem 2	/10
Problem 3	/15
Problem 4	/10
Problem 5 (Bonus)	/5
Total Score	/45
Midterm Grade	

Date: July 15, 2004.

Problem 1.

(a) Compute $g = \gcd(2004, 99)$.

(b) For g above, find integers $x, y \in \mathbb{Z}$ such that

$$2004x + 99y = g.$$

Problem 2. Let $a, b \in \mathbb{Z}_{>1}$ satisfy $a^3 = b^2$. Show that there exists a $d \in \mathbb{Z}$ such that $a = d^2$ and $b = d^3$.

Problem 3.

(a) Find a solution $x \in \mathbb{Z}/27\mathbb{Z}$ to the congruence
$$x^2 - 7x \equiv 6 \pmod{27}.$$

(b) How many distinct solutions $x \in \mathbb{Z}/243\mathbb{Z}$ are there to the congruence
$$x^2 - 7x \equiv 6 \pmod{243}?$$

Problem 4. What is the smallest prime divisor of $n = 365^{2004} + 94$?

Problem 5 (Bonus). Let $\alpha = \arctan(7/2)$. Show that $\sin(\alpha)$ is irrational.