

**ERRATA:**  
***ℓ-ADIC IMAGES OF GALOIS FOR ELLIPTIC CURVES OVER  $\mathbb{Q}$***

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 WITH JOHN VOIGHT

This note gives errata for the article *ℓ-adic images of Galois for elliptic curves over  $\mathbb{Q}$*  [1].

- (1) Page 9, line 1: “ $hgu \mapsto hg\chi_N(\sigma)u$ ” should be “ $HgU(N) \mapsto Hg\chi_H(\sigma)U(N)$ ”.
- (2) Page 9, line 1: “[DR73, IV-5.3]” should be “[DR73, VI-5.3]”.
- (3) Page 9, line 7: “that contain a conjugate of” should be “fixed by the action of”.
- (4) (5.1): “ $j(E) \in k$ ” should be “ $j(E) \in \mathbb{F}_q$ ”.
- (5) Page 19, line -3: “322102” should be “146410”.
- (6) Page 23, line 19: An error was made in the computation of the image on the  $j$ -line for the two points in the base locus of  $X_H \rightarrow \mathbb{P}^1$ . The  $j$ -invariants should be 1728 and  $-884736000$ .
- (7) Page 28, lines 24-26: The five additional points in  $X'_H(\mathbb{Q}(\zeta_3))$  are four CM points with discriminant  $-4$ , and one with  $j$ -invariant  $2^3 \cdot 5^3 \cdot (1 - \zeta_3)^6 \cdot (5 + 3\zeta_3)^{-27} \cdot (27 + 13\zeta_3)^3 \cdot (54 + 49\zeta_3)^3 \cdot (227 + 173\zeta_3)^3$
- (8) Page 28, line 31: The CM points found have discriminants  $-4, -7, -16, -19, -28, -43, -67$  and  $-163$ .
- (9) Table 9, section 11: The totals listed in the “SW” line of the first part of Table 9 are incorrect due to double counting 312435 elliptic curves of prime conductor that are included in two parts of the Stein–Watkins database. The Stein–Watkins database contains all elliptic curves over  $\mathbb{Q}$  of absolute discriminant at most  $10^{12}$  that have  $c_6$ -invariant of absolute value at most  $1.44 \cdot 10^{12}$  whose conductor  $N$  meets either of the following criteria (which are not mutually exclusive): (1)  $N \leq 10^8$ , (2)  $N \leq 10^{10}$  is prime.

The corrected counts in the first part of the table are:

SW 35585316 3671438 181222 43966 2048 7444 368 1024 97776484 136789294

The counts in the second part of the table are the same (none of the double-counted curves are non-surjective at more than one prime).

- (10) Proposition A.6: the final phrase should be “conductor dividing  $N$ ” (not “character of modulus  $N$ ”).
- (11) [SW02]: the book title “Algorithmic Number Theory Symposium (ANTS I)” should be “Algorithmic Number Theory 5th International Symposium (ANTS V)”.

REFERENCES

- [1] Jeremy Rouse, Andrew V. Sutherland, and David Zureick-Brown, *ℓ-adic images of Galois for elliptic curves over  $\mathbb{Q}$* , appendix with John Voight, Forum Math. Sigma **10** (2022), e62.

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