

**ERRATA:**  
**COMPUTING CLASSICAL MODULAR FORMS**

ALEX BEST ET AL.

This note gives errata and addenda for the article *Computing classical modular forms* [1].

1. ERRATA

- (1) Theorem 5.1.2: Replace  $S_k(\Gamma_1(n); \mathbb{Q})$  with  $S_k(\Gamma_1(N); \mathbb{Q})$ .
- (2) Section 8.3: Replace  
 For a newform  $f$  with trivial character, its Fricke eigenvalue is *minus* the sign of the functional equation of its  $L$ -function, and each  $W_q$ -eigenvalue is the sign of a certain local functional equation.  
 with  
 For a newform of weight  $k$  and trivial character, the Fricke eigenvalue  $\epsilon$  is related to the sign  $\varepsilon$  that appears in the functional equation (9.1.3) via  $\epsilon = (-1)^{k/2}\varepsilon$ , see Miyake [70, Cor. 4.3.7]. Each  $W_q$ -eigenvalue is similarly related to the sign of a certain local functional equation.
- (3) Conjecture 8.5.1: Missing new subspace. Replace statement with  
 For all  $k \geq 2$ , the space  $S_k^{\text{new}}(\Gamma_0(2))$  decomposes under the Atkin–Lehner operator  $W_2$  into Hecke irreducible subspaces of dimensions  $\lfloor d/2 \rfloor$  and  $\lceil d/2 \rceil$  where  $d := \dim_{\mathbb{C}} S_k^{\text{new}}(\Gamma_0(2))$ . In the discussion that follows, replace the reference Kimball [64] with the reference to [Kimball Martin, *Refined dimensions of cusp forms, and equidistribution and bias of signs*, J. Number Theory **188** (2018), 1–17.].

2. ADDENDA

- (1) Section 8.5: Strike “However, we observed behavior analogous to the Maeda conjecture in weight 1 up to weight  $k \leq 400$ , with the additional prediction that the Atkin–Lehner operator splits the space as evenly as possible.” Replace with “However, we observed behavior analogous to the Maeda conjecture in weight 1 up to weight  $k \leq 400$ . The Atkin–Lehner operator  $W_2$  splits the space as evenly as possible, and the  $W_2$ -eigenspaces appear to always be irreducible.”
- (2) We can prove the observation following Conjecture 8.5.1. Replace the text up to Question 8.5.3 with the following:  
 The dimensions in the corollary follow from Martin [Thm. 2.2] (Kimball Martin, *The basis problem revisited*, arXiv:1804.04234v2, 2019), which implies that for even weights  $k > 2$  we have

$$\dim S_k^{\text{new}}(\Gamma_0(2))^+ - \dim S_k^{\text{new}}(\Gamma_0(2))^- = \begin{cases} 0 & k = 4, 6 \pmod{8}, \\ (-1)^{k/2} & k \equiv 0, 2 \pmod{8}, \end{cases}$$

is only the irreducibility of the eigenspaces that is conjectural. The factor  $(-1)^{k/2}$  in (8.5.2) appears as 1 in [64, Thm. 2.2] because there the Atkin–Lehner operator follows the convention of Diamond–Shurman [39, p. 209], which includes a factor of  $(-1)^{k/2}$ , while we are following the convention of Miyake [70], which does not include this factor. One can find similar formulas for  $\dim S_k^{\text{new}}(\Gamma_0(N))^+ - \dim S_k^{\text{new}}(\Gamma_0(N))^-$  for any squarefree  $N$  in Martin, in which case they are a linear function of the class number  $h(-4N)$ . For

general  $N > 4$  not of the form  $M^2, 2M^2, 3M^2, 4M^2$  with  $M$  squarefree, we refer the reader to Helfgott [Harald A. Helfgott, *Root numbers and the parity problem*, Ph.D. Thesis, Princeton University, 2003, pp. 266-267].

#### REFERENCES

- [1] Alex J. Best, Jonathan Bober, Andrew R. Booker, Edgar Costa, John Cremona, Maarten Derickx, Min Lee, David Lowry-Duda, David Roe, Andrew V. Sutherland, and John Voight, *Computing classical modular forms*, Arithmetic Geometry, Number Theory, and Computation, eds. Jennifer S. Balakrishnan, Noam Elkies, Brendan Hassett, Bjorn Poonen, Andrew V. Sutherland, and John Voight, Simons Symp., Springer, Cham, 2021, 131–213.