ERRATA AND ADDENDA: *ON COMPUTING BELYI MAPS*

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This note gives some errata and addenda for the article *On computing Belyĭ maps* [3]. The authors thank Sam Schiavone.

Errata

(1) Section 1, (D3): To be more precise, replace with

> O is a cyclic orientation of the edges around every vertex. That is, $O = (O_0, O_1)$ is a pair of permutations in $\mathrm{Sym}(E)$ with the property that two edges e, e' are in the same orbit under O_0 if and only if the vertices $v_0(e), v_0(e')$ colored 0 are equal, and the same with the orbit under O_1 for the vertices colored 1.

Then replace the first sentence with "Concretely, a cycle of O_0 with common vertex v colored 0 describes the result of rotating the edges around v counterclockwise, and the same with O_1. For some examples, see e.g. Couveignes–Granboulan [1, p. 15–16]."

(2) Section 2, "The ASD differentiation trick": Elkies [2, Footnote 4] notes that the use of differentiation in this context goes back at least as far as 1956, appearing on the Putnam exam (and thus predating Atkin and Swinnerton-Dyer).

(3) Section 6, paragraph 3, "In particular, the CM factors of the Jacobian factors of the Galois Belyĭ curves are essentially known; they come from Fermat curves": This is not correct; this only is only true for the factors of the Jacobian that are one-dimensional subrepresentations of the automorphism group, as happens in the case of abelian automorphism groups but not in general. In Section 6.5 of the article of Wolfart that is referred to, Hurwitz curves are constructed that are not of CM type and that therefore cannot come from Fermat curves.

(4) (7.7): Should be $\# \mathrm{Aut}_G(f, i)$ in the formula.

(5) Section 8, Simplification: When considering denominator and numerator, make it clear that it is Belyĭ maps on hyperelliptic curves that are considered throughout.

Addenda

(1) Section 4, after (4.5): Elkies notes that one can often generate spaces of modular forms or more simply compute a Hauptmodul for *congruence* subgroups of $\mathrm{SL}_2(\mathbb{Z})$ (and the corresponding Belyĭ maps) by using other techniques, such as eta quotients or theta functions associated to integral

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quadratic forms, as well as their images under differentiation and Hecke operators.

References

[1] Jean-Marc Couveignes and Granboulan, *Dessins from a geometric point of view*, in *The Grothendieck theory of dessins d'enfants*, London Math. Soc. Lecture Note Ser., vol. 200, Cambridge University Press, 1994, 79–113.
[2] Noam D. Elkies, *ABC implies Mordell*, Internat. Math. Res. Notices **1991**, no. 7, 99–109.
[3] Jeroen Sijsling and John Voight, *On computing Belyi maps*, accepted to Publ. Math. Besancon.