ERRATA: *EXPLICIT METHODS FOR HILBERT MODULAR FORMS*

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This note gives some errata for the article *Explicit methods for Hilbert modular forms* [1]. Thanks to Benjamin Breen, Nuno Freitas, and Dino Lorenzini.

(1) Page 137, paragraph after (1.4), "then (1.1) is equivalent to": This is not correct (it is OK only for $k = 2$), even with the algebraic normalization. Statement (1.1) is equivalent to
$$f(\gamma z)(d(\gamma z))^{k/2} = f(z)(dz)^{k/2}$$
but if k is odd one must worry about what branch of the square root to take.

(2) Page 137, paragraph after (1.4), "(Because of our normalization...)": This statement is probably confusing, as the term *local system* in this context refers to vector-valued forms, while we are talking about line bundles. Instead, one should work with line bundles, and consider the action of $\Gamma_0(N)$ on $\mathcal{H} \times \mathbb{C}$ by
$$(z, v) \mapsto \left(\gamma z, \frac{j(\gamma, z)^k}{(\det \gamma)^{k-1}} v\right)$$
for $\gamma \in \Gamma_0(N)$ and $(z, v) \in \mathcal{H} \times \mathbb{C}$, which gives rise to a line bundle on $X_0(N) = \Gamma_0(N)\backslash\mathcal{H}$ whose sections are modular forms of weight k. These agree with differential forms up to a twist by a power of the determinant; our normalization is more convenient in algebraic contexts, but in any case the Hecke module structure is the same.

(3) Page 140, line after (2.4), "then (3.3) is equivalent to": Should be "(2.2)", not (3.3).

(4) Page 140, after (2.5), "we may write $\mathfrak{n} = \nu\mathfrak{d}^{-1}$ for some $\nu \in \mathfrak{d}_+$": Should be $\mathfrak{n} = \nu\mathfrak{d}$. We are taking $\nu \in \mathfrak{d}_+^{-1}$, so
$$\nu\mathfrak{d} \subseteq \mathfrak{d}^{-1}\mathfrak{d} = \mathbb{Z}_F$$
giving the desired sum over integral ideals $\mathfrak{n} = \nu\mathfrak{d}$.

(5) Page 145, (3.5), line -4, "Let $\mathfrak{p}$ be a prime of $\mathbb{Z}_F$": Need $\mathfrak{p} \nmid \mathfrak{D}\mathfrak{N}$.

(6) Page 146, last line, "extends by linearity to all of $S_2^B(\mathfrak{N})$": Not needed: definition (3.8) already makes sense in all cases.

(7) Page 148, line 9, "let $w_i = \#(\mathcal{O}_i/\mathbb{Z}_F^\times)$": should be $e_i =$.

(8) Example 6.3, line -4: Should be "the isogeny theorem of Faltings".

(9) Example 6.4, line -6: Should be "$\mathbb{F}_9$", not F_9.

(10) Example 6.4, line -5: $\mathfrak{p}$ should be $\mathfrak{N}$.

(11) Example 6.8, line -4: should say "Indeed, Ogg showed that the abelian surface $J_1(13)$ has a rational point of order 19 [3, p. 225] and Mazur and

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Tate [2] showed that $J_1(13)$ twisted by $\mathbb{Q}(\sqrt{13})$ is a product of two elliptic curves."

(12) Lemma 7.11, "second one": Possibly confusing, should be "second variable".

(13) Before (7.27), "Suppose that $[\mathfrak{n}] = [\mathfrak{a}\mathfrak{d}^{-1}]$ ... be such that $\mathfrak{n} = \nu\mathfrak{a}\mathfrak{d}^{-1}$": Should be $[\mathfrak{n}] = [\mathfrak{a}\mathfrak{d}^{-1}]^{-1}$ and $\mathfrak{n} = \nu(\mathfrak{a}\mathfrak{d}^{-1})^{-1} = \nu\mathfrak{d}\mathfrak{a}^{-1}$.

References

[1] Lassina Dembélé and John Voight, *Explicit methods for Hilbert modular forms*, Elliptic curves, Hilbert modular forms and Galois deformations, Birkhauser, Basel, 2013, 135–198.

[2] B. Mazur and J. Tate, *Points of order* 13 *on elliptic curves*, Invent. Math. **22** (1973/74), 41–49.

[3] A. P. Ogg, *Rational points on certain elliptic modular curves*, Analytic number theory, Proc. Sympos. Pure Math., Vol. XXIV, Amer. Math. Soc., Providence, RI, 1973, 221–231.