ERRATA: *SMALL ISOSPECTRAL AND NONISOMETRIC ORBIFOLDS OF DIMENSION* 2 *AND* 3

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This note gives some errata for the article *Small isospectral and nonisometric orbifolds of dimension* 2 *and* 3 [2]. The authors thank Aurel Page and Alex Bartel.

Errata

(1) Proposition 2.5 can be strengthened as follows.

Proposition 2.5.
(a) [Vignéras [3, Corollaire 5]] *Suppose that for all $g \in G$, the weight of g^G over Γ is equal to the weight of g^G over Γ'. Then Γ and Γ' are representation equivalent.*
(b) [Gordon–Mao [1, Theorem A]] *Suppose that $G = \mathrm{SO}(d,1)$ for $d \geq 1$ and that $\Gamma, \Gamma' \leq G$ are cocompact. Then the converse to* (a) *holds: Γ and Γ' are representation equivalent if and only if for all $g \in G$, the weight of g^G over Γ is equal to the weight of g^G over Γ'.*

Proof. Part (a) of Proposition 2.5 is proven by Vignéras as a direct consequence of the Selberg trace formula.

For part (b), we refer to Gordon–Mao [1, Theorem A, Lemma 1.1]. Our hypothesis that $G = \mathrm{SO}(d,1)$ implies that the quotients $\Gamma\backslash G/K$ and $\Gamma'\backslash G/K$ for $K = \mathrm{SO}(d)$ are compact, hyperbolic orbifolds. The rest of their proof can be applied verbatim, restoring the hypothesis that the groups are representation equivalent. □

(2) Remark 2.6: To clarify, change first sentence to "In fact, in dimension 2, a converse to Theorem 2.1 holds". The relevant converse in Proposition 2.5 is above, part (b).
(3) End of the proof of Theorem 6.4: Expand to "else employ Theorem 2.19 to exhibit a selectivity obstruction which precludes the possibility that the groups are representation equivalent: by Proposition 2.5(b), there is a conjugacy class corresponding to the selective order that embeds in one group and not the other."
(4) Before (4.14), "Lemma lem:neq5": should be Lemma 4.13.
(5) Theorem D, Example 6.3: The example is not a manifold. In Lemma 5.1, we characterized when there are no torsion elements in a group Γ^1 coming from norm 1 units, but this example concerns a larger group Γ^+. We would have needed to check in addition that for every totally positive unit $u \in \mathrm{nrd}(\Gamma)$ that $F(\sqrt{-u})$ does not embed in B. In fact, there is such an

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embedding for the example, and consequently there are nontrivial 2-torsion elements in Γ. (Such a nontrivial 2-torsion element can be given explicitly.)

Theorem D did not claim to find the smallest example, so the infraction is minor—in particular, the other results in the paper are unaffected, and the 2-manifold example remains correct.

We can find an example to replace Theorem D which is only a bit bigger as follows. Let $F = \mathbb{Q}(t)$ be the quintic field with discriminant -43535 and defining polynomial $x^5 - x^4 + 3x^3 - 3x + 1$. Let $B = \left(\dfrac{3t^3 - 2, -13}{F}\right)$, so that B is ramified at the three real places of F and the prime ideal $\mathfrak{p} = (t^4 - t^3 + 3t^2 - t - 2)$ of norm 13. The algebra B contains two conjugacy classes of maximal orders, does not admit an embedding of a quadratic cyclotomic extension of F and obviously exhibits no selectivity as selectivity can only occur in quaternion algebras unramified at all finite primes. Thus the hyperbolic 3-manifolds associated to two non-conjugate maximal orders will be isospectral. The volume of these isospectral manifolds is $51.024566\ldots$ So in Theorem D, $39.2406\ldots$ should be replaced with this larger volume.

Consequently, the remarks on Theorem D at the end of the paper must also be adjusted upwards, but they remain as daunting as they were before.

(6) Theorem 6.4: This should read as in the statement of Theorem C, namely, "The smallest volume of a *representation equivalent*-nonisometric pair".

References

[1] C. Gordon and Y. Mao. Comparisons of Laplace spectra, length spectra and geodesic flows of some Riemannian manifolds. *Math. Res. Lett.*, 1(6):677–688, 1994.

[2] Benjamin Linowitz and John Voight, *Small isospectral and nonisometric orbifolds of dimension* 2 *and* 3, Math. Z. **281** (2015), no. 1, 523–569.

[3] Marie-France Vignéras, *Variétés riemanniennes isospectrales et non isométriques*, Ann. of Math. (2) **112** (1980), no. 1, 21–32.