ERRATA: *HEEGNER POINTS AND SYLVESTER'S CONJECTURE*

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This note gives errata and addenda for the article *Heegner points and Sylvester's conjecture* [1]. The authors would like to thank Jeong Keunyoung.

Errata

(1) We did not define the action of multiplication by ω in section 3, which on $E : y^2 = x^3 + 1$ we are taking to be

$$\omega(x, y) = (\omega x, y), \quad \text{for } (x, y) inE(K)$$

This is not to be confused with the action of ω on $E_n : x^3 + y^3 = nz^3$ in section 2.3.

(2) At the end of §3.4, we wrote that $E_{\text{tors}}(L) = \{O, (0, \pm 1)\}$, but this is false: in fact, already

$$E_{\text{tors}}(\mathbb{Q}) = \{O, (0, \pm 1), (-1, 0), (2, \pm 3)\} \simeq \mathbb{Z}/6\mathbb{Z}.$$

To fix this, we argue as follows. Let $S \in E[m](L)$ satisfy (7) with $m \in \mathbb{Z}_{\geq 1}$. Multiplying by 3 and noting $3(0, -1) = O$ gives $3S^\sigma = (3S)^\sigma = \omega(3S)$, so $3S$ twists to a point in $E_{2p}[m](K)$. The curve E_{2p} has additive reduction at 3, and so the component group $\#\Phi(\mathbb{F}_3) \leq 4$; thus the twist of $12S$ belongs to the identity component of the special fiber (isomorphic to $\mathbb{F}_3$), so $36S = O$. Factoring the 4- and 9-division polynomials on E_{2p} gives in fact

$$S \in E_{\text{tors}}(K) = \langle(-\omega, 0), (2, 3)\rangle \simeq \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z}.$$

But then $S^\sigma = S$, so from (7) we find $(1 - \omega)S = (0, -1)$, and this is a contradiction: we can check each of the 12 torsion points, or note that $(0, -1)$ is not divisible by $1 - \omega$ in $E_{\text{tors}}(K)$—the $(1 - \omega)$-division points of $(0, -1)$ in fact belong to the field $K(\sqrt[3]{2})$.

References

[1] Samit Dasgupta and John Voight, *Heegner points and Sylvester's conjecture*, Arithmetic Geometry, Clay Math. Proc., vol. 8, Amer. Math. Soc., Providence, 2009, 91–102.

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