Dartmouth-UVM Math Day

10 February 2024

9:00-9:55 a.m. coffee (upstairs in Kemeny 300)

9:55-10:00 a.m. welcome (Haldeman 041)

10:00-10:50 a.m. Dimitrios Giannakis (Dartmouth): Quantum information and data science for modeling classical dynamics

Over the past three decades, a fruitful approach for analysis and data-driven modeling of dynamical systems has been to consider the action of (nonlinear) dynamics in state space on linear spaces of observables. These methods leverage the linearity of the associated evolution operators, namely the Koopman and transfer operators, to carry out tasks such as mode decomposition, forecasting, and uncertainty quantification using linear operator techniques. Mathematically, the operator-theoretic approach has close connections with representations of nonlinear transformations (the state space dynamics) into spaces of functions (the observables) with a commutative algebraic structure.

In this talk we discuss generalizations of this framework to the setting of non-commutative algebras of operators using ideas from quantum information science. Central to our approach is a representation of observables and probability densities through multiplication operators and density operators (quantum states), respectively. Using these objects, and the dynamical operators governing their evolution, we formulate two common problems in dynamical systems modeling, namely data assimilation and dynamical closure, in an operator-theoretic language. We discuss how the operator-theoretic approach leads to structure-preserving computational schemes (e.g., positivity-preserving function approximation) which are also amenable to data-driven implementation using kernel methods for operator approximation. We present applications to data assimilation of the El Nino Southern Oscillation of the climate system and subgrid-scale modeling in multiscale systems. In a second part of the talk, we discuss implementations of these methods on quantum computers.

11:10-11:30 p.m. Will Thompson (UVM): Inferring interaction kernels in stochastic opinion dynamics models

The field of opinion dynamics studies the evolution of opinions or beliefs on social networks. Many opinion dynamics models feature a kernel — a function that dictates how a node updates its opinion based on neighboring nodes. We present preliminary work demonstrating how kernel-based opinion dynamics models could be learned from data. This work studies asynchronous, discrete-time stochastic models of opinion dynamics parameterized by such kernels.

We implement an expectation-maximization (EM) algorithm that successfully recovers the ground truth kernel when provided a network and a time series of node opinions. Here opinions are molded as a real-valued quantity that represents an individual's belief on some spectrum. Our inference approach performs well across various kernels in the literature. Furthermore, we adapt our EM algorithm using gradient-free optimization methods to infer non-differentiable kernels such as the bounded confidence kernel successfully. Finally, we present a non-parametric adaptation of our algorithm, which uses a multilayer perceptron to learn kernel shapes. This allows for the effective non-parametric learning of kernel shapes, broadening our algorithm's applicability.

We demonstrate that our refined technique can accurately reconstruct a diverse spectrum of kernels. For each kernel type, we report the identifiability of various parameter values and show that our methods successfully recover ground truth across a wide range of parametrizations. This preliminary work proves the feasibility of kernel inference methods and suggests that such methods could be used to fit models of opinion dynamics to empirical data in the future. Significantly, our approach offers dual capabilities: it can rigorously test the explanatory power of hypothesized kernels and also learn kernels from data in a non-parametric manner. Our analysis shows that our algorithm is a powerful tool for inferring stochastic opinion dynamics models from time series data regardless of the form and differentiability of the kernel.

11:35-11:55 p.m. Jesse Franklin (UVM): Geometry of Drinfeld modular forms

We introduce the Drinfeld setting, Drinfeld modular curves and forms. Our aim is to explain what it means for Drinfeld modular forms to be sections of a line bundle on a Drinfeld modular curve. We mention some historical motivation and context for this result and discuss some applications and future work as time permits.

12:00-12:20 p.m. Andrew Hanlon (Dartmouth): Integrality of mirror maps

One of the early predictions of mirror symmetry is that mirror maps, which identify moduli spaces of mirror Calabi-Yau varieties, have integral coefficients when expanded as a series in appropriate coordinates. This conjecture has been proved in various settings using techniques of analytic number theory. I will discuss joint work with Ganatra, Hicks, Pomerleano, and Sheridan that shows that the integrality of mirror maps conjecture follows from an arithmetic refinement of homological mirror symmetry and proves new cases of the conjecture.

12:30-1:45 p.m. lunch (Haldeman 031)

1:45-2:05 p.m. Anton Hilado (UVM): Special loci on moduli spaces of abelian varieties with complex multiplication

Moduli spaces of abelian varieties possess very interesting geometry. For instance, they may contain subvarieties which are themselves moduli spaces of what may be viewed as different abelian varieties. The study of the intersections of these special subvarieties with other "special" loci such as the loci where the abelian varieties have certain endomorphism structure is a generalization of the theory of singular moduli studied by Gross and Zagier. The work of Lauter and Yang (and later Lauter and Viray) applies the theory to study certain invariants of cryptographic interest for genus 2 curves. In this talk we will describe ongoing work on a genus 3 version of these pioneering works.

2:10-2:30 p.m. Mariah Boudreau (UVM): Temporal and probabilistic comparisons of epidemic forecasts

Forecasting disease spread is a critical tool to help public health officials design and plan public health interventions. However, the expected future state of an epidemic is not necessarily well defined as disease spread is inherently stochastic, contact patterns within a population are heterogeneous, and behaviors change. In this work, we use time-dependent probability generating functions (PGFs) to capture these characteristics by modeling a stochastic branching process of the spread of a disease over a network of contacts in which public health interventions are introduced over time. To achieve this, we define a general transmissibility equation to account for varying transmission rates (e.g. masking), recovery rates (e.g. treatment), contact patterns (e.g. social distancing) and percentage of the population immunized (e.g. vaccination). The resulting framework allows for a temporal and probabilistic analysis of an intervention's impact on disease spread, which match continuous-time stochastic simulations that are much more computationally expensive. To aid policy making, we then define several metrics over which temporal and probabilistic intervention forecasts can be compared: Looking at the expected number of cases and the worst-case scenario over time, as well as the probability of reaching a critical level of cases and of not seeing any improvement following an intervention. Given that epidemics do not always follow their average expected trajectories and that the underlying dynamics can change over time, our work paves the way for more detailed short-term forecasts of disease spread and more informed comparison of intervention strategies.

2:35-2:55 p.m. Eran Assaf (Dartmouth): Canonical rings of stacky surfaces

How do algebraic varieties embed in projective space? Varieties of general type admit a canonical embedding, and for the case of curves it was classically studied and one can specify the degrees of generators and relations for such an embedding. The landscape of surfaces is more complicated and, although it has been studied for decades, the situation is not as clear. In this talk, we will present what is known for surfaces, generalize it to certain algebraic stacks of dimension 2, and present an application for Hilbert modular surfaces.

3:00-3:30 p.m. coffee break

3:30-3:50 p.m. Calum Buchanan (UVM): Saturation numbers: double stars and a general lower bound

Let *H* be a graph with at least one edge. A graph *G* is said to be *H*-saturated if *G* does not contain a subgraph isomorphic to *H*, but the addition of any extra edge to *G* creates such a subgraph. In 1964, Erdős, Hajnal, and Moon first studied the minimum number of an edges in an H-saturated graph of a given order when *H* is a clique. A nontrivial lower bound on this "saturation number" for a general graph *H* was not seen until 2022, due to Cameron and Puleo. This talk will focus on a new lower bound on the saturation number, with a strengthening when H is triangle-free. We will examine its effectiveness on the class of trees with diameter 3, also known as double stars. This talk is based on joint work with Puck Rombach.

4:00-4:30 p.m. Lightning talks!

- Taylor Dupuy (UVM): My favorite orders in Clifford algebras

I am going to show pictures of fundamental domains and show some stuff.

- Tristan Phillips (Dartmouth): Average ranks of elliptic curves over number fields

Understanding rational solutions to polynomial equations is a central problem in number theory. In the case of polynomials in two variables, this is closely related to the problem of finding rational points on plane curves. One of the simplest questions to ask about such a curve is whether it has infinitely many or finitely many rational points. This question has been satisfactorily answered in all cases except for a special class of curves, known as elliptic curves. In this talk I will briefly discuss how techniques from Diophantine geometry, *o*-minimal geometry, and modular forms can come together to tell us something about the statistical behavior of rational points on elliptic curves over number fields.

- James Schmidt (Dartmouth): Symmetric chain decomposition of *N(m,n)*

A recent paper found an algorithm to create a symmetric chain decomposition of the poset *N(m,n)*. However, the paper is essentially illegible. Therefore, I made a better version of it, and I’ll show the core algorithm in five minutes and give some reasons why it works.

- Longmei Shu (Dartmouth): Rock-paper-scissors game and male lizards

Over a 6 year period, the population of a side-blotched lizard oscillates among domination of three different types of male lizards. The ones with an orange throat have a large territory and multiple females in their territory. The ones with a blue throat have a small territory and one female in their territory. The third type of lizards have a yellow throat and look like females. They don't have territories and sneak around. We will use rock-paper-scissors game to model this process.

- Tobias Timofeyev (UVM): The role of graph partitions for cluster synchronization in weighted graphs

We explore the relationship between graph partitions, graph Laplacian eigenvectors, and cluster synchronization in the Kuramoto model. Almost equitable partitions (AEPs) have been linked to cluster synchronization of oscillatory systems. Using spectral properties of AEPs, we recast these findings in terms of graph Laplacian eigenvectors. Additionally, this description of the problem allows us to define a relaxation of an AEP we call a quasi equitable partition (QEP). We find these partitions allow for qualitatively similar clustering behavior as with AEPs.

- Pete Winkler (Dartmouth): The home-field paradox

A curious sports paradox will be presented for general amusement.

4:45-5:35 p.m. Spencer Backman (UVM): Graphs and matroids: the tropical way

Tropical geometry is a piecewise linear version of classical algebraic geometry motivated by the study of degenerations. The development of tropical geometry over the past two decades has uncovered deep connections with many other fields. Graphs and matroids are two fundamental objects of study in combinatorics, and from the point of view of tropical geometry, they correspond to curves and linear spaces, respectively. The emergence of this perspective has been fruitful for both algebraic geometry and combinatorics. In this talk, I will give an overview of the tropical perspective on graphs and matroids, and in passing describe a couple of my own contributions to these topics.